Phase-Transition Modulated Mixing in the Mantle of the Earth [and Discussion]

W. R. Peltier, R. K. O'Nions, J. H. Davies and S. K. Runcorn

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Phase-transition modulated mixing in the mantle of the Earth

By W. R. Peltier

Department of Physics, University of Toronto, 60 St George St., Toronto, Ontario, Canada M5S 1A7

Recently published *a priori* analyses of the mantle convection process demonstrate that the influence of the endothermic phase transformation at 660 km depth could be profound and such as to enforce a strongly intermittent style of circulation. In this phase-transition modulated regime, mass transfer across the 660 km horizon is inhibited, leading to the formation of an internal thermal boundary layer that straddles the phase change interface. This thermal boundary layer is itself episodically disrupted by convective instability, leading to the development of intense 'avalanches' of cold material from the transition zone into the lower mantle. The consequences of such a circulation regime to the understanding of trace element geochemistry and the supercontinent cycle may be extremely important. Equally important are the implications for mantle viscosity, since a circulation of this type would appear to allow reconciliation of a wide range of geodynamic data with a single viscosity model. The relevant data include convection related observables such as the aspherical geoid and postglacial rebound related observables such as relative sea level histories and certain anomalies in Earth rotation, including the so-called 'non-tidal' acceleration of rotation and the ongoing 'wander' of the rotation pole towards Greenland at a rate near 0.95 degrees per million years.

1. Introduction

The regulation of the thermal regime of the deep interior of the planet, by convective circulation in the iron–magnesium–silicate mantle that envelops its liquid iron outer core, involves a process of considerable subtlety. This process transports heat from the core–mantle boundary to the Earth's surface, thereby controlling the rate of core cooling and thus the 'lifetime' of the magnetic field. Several recent analyses of the impact upon the efficiency of this transport, due to the pressure-induced phase transitions that bracket the mantle transition zone (Machetel & Weber 1991; Peltier & Solheim 1992; Tackley *et al.* 1993), have suggested that this effect may be extremely important. Although the shallower of these transitions that occurs at 400 km depth, due to the transformation of the olivine constituent ($(\mathrm{Mg}_{(1-x)}\mathrm{Fe}_x)_2\mathrm{SiO}_4$ with $x \approx 0.1$) into the spinel phase (Ringwood & Major 1966; Akimoto & Fujisawa 1966), has some impact upon the flow, the paper by Peltier & Solheim (1992) demonstrated that mixing is most profoundly influenced by the deeper horizon. This further transformation of phase is one in which spinel is converted into a mixture of (Mg, Fe)O pagnesiowüstite and $(\mathrm{Mg,Fe})\mathrm{SiO}_3$ perovskite via the reaction

$$(\mathrm{Mg,Fe})_2\mathrm{SiO}_4 \rightarrow (\mathrm{Mg,Fe})\mathrm{O} + (\mathrm{Mg,Fe})\mathrm{SiO}_3$$

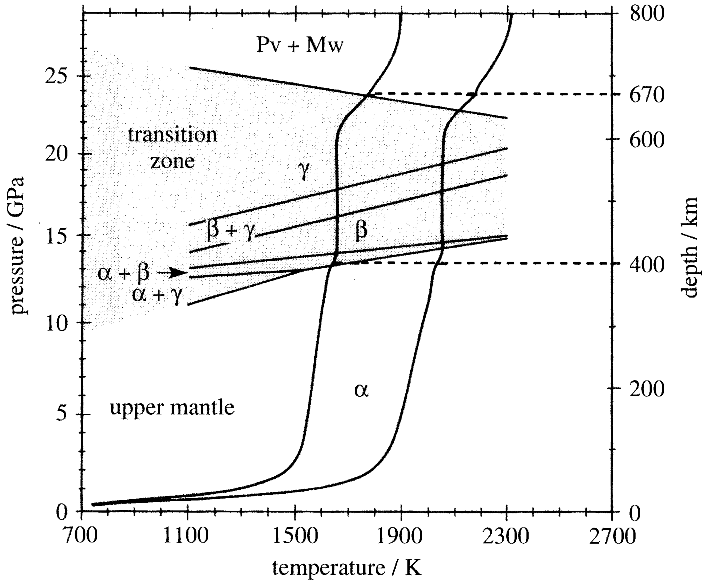

Figure 1. Phase diagram for the Mg_2SiO_4–Fe_2SiO_4 component of the mineralogy of the mantle. The shaded region in the P–T plane is the transition region which is bracketed below (at low pressure) by the olivine–spinel ($\alpha \rightarrow \beta$) phase transformation and above by the spinel–perovskite + magnesiowüstite transition. The two solid curves superimposed upon the phase diagram are time and azimuthally averaged 'geotherms' from the *a priori* simulations of the mantle convection process to be discussed in this paper. Both geotherms are from simulations at $Ra = 10^7$, the high-temperature geotherm being derivative of a model with 50% heating from within and 50% heating from below and the low-temperature geotherm of a model heated entirely from below.

Key to understanding the dynamical impact of these two phase transformations on convective mixing is the Clapeyron slope $\mathrm{d}p/\mathrm{d}T$ of each transition. The shallower transformation is exothermic, with $\mathrm{d}p/\mathrm{d}T$ between $+2$ and $+3\ \mathrm{MPa\,K^{-1}}$ (Akaogi *et al.* 1989; Katsura & Ito 1989) and a divariant phase loop thickness of 18 ± 5 km (Akaogi *et al.* 1989). The deeper horizon is endothermic, with $\mathrm{d}p/\mathrm{d}T = -4 \pm 2\ \mathrm{MPa\,K^{-1}}$ according to Ito *et al.* (1989, 1990). The phase loop thickness for this transition is extremely small according to Ito & Takahashi (1989), whose measurements constrain it to be not more than 4 km. The basic characteristics of the phase diagram for the olivine constituent of the mantle in the p–T plane are shown in figure 1.

The plan in this paper is to describe the properties of the mantle convective circulation that is expected to develop when the full influence of these phase transformations is included and, furthermore, to discuss the implications of these results to the understanding of deep mantle physical properties, principally the momentum diffusivity, but also trace element geochemical data that may be invoked to constrain radial mixing. To this end, § 2 is devoted to the description of a model of the mantle convection process that has been specifically developed to enable accurate analyses of phase-transition modulated mixing to be performed at high Rayleigh number. The main results that have been obtained to date through the application of this model are thereafter discussed in § 3 where they are also placed in the context of both earlier and other recent investigations. Although the simulations of mantle mixing that have been performed with this model have all been based upon the assumption that

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the viscosity of the mantle could be assumed constant, I will argue in § 4 that the thermal structure of the intermittently layered mixing regime that is characteristic of these flows suggests that a certain equally characteristic variation of viscosity should exist across the endothermic phase-transition horizon. In the remainder of the paper it is suggested that the existence of this characteristic structure would help enable one to reconcile the sharply discrepant estimates of the radial variation of mantle viscosity that have been inferred on the basis of seismic tomography-based models of the non-hydrostatic geoid, on the one hand, and models of glacial isostatic adjustment on the other. A brief summary of the paper is offered in § 5 along with a number of general conclusions.

2. Thermal convection with phase transitions: a model

In the limit of infinite Prandtl number $Pr = \nu/\kappa$, with ν and κ the diffusivities for momentum and heat, respectively, and, in the anelastic approximation (Batchelor 1954; Jarvis & McKenzie 1981), the laws of mass, momentum and internal energy conservation may be written, respectively, in the following non-dimensional forms:

$$\nabla \cdot (\rho_{\mathrm{r}} \boldsymbol{u}) = 0, \tag{2.1}$$

$$0 = -\rho g \hat{r} - \nabla p + \alpha_0 T_{\mathrm{c}} \{\nabla \times \nabla \times \boldsymbol{u} + \tfrac{4}{3} \nabla \cdot \boldsymbol{u}\}, \tag{2.2}$$

$$\begin{aligned}\frac{\mathrm{D}T}{\mathrm{D}t} + \tau u_{\mathrm{r}}(T + T_0) = \frac{\kappa}{Ra}\left\{\nabla^2 T + \frac{1}{\kappa}\frac{\mathrm{d}\kappa}{\mathrm{d}r}\frac{\partial T}{\partial r}\right\} + \frac{\mu}{c_{\mathrm{p}}\, Ra} + \frac{\tau_0 \Psi}{\rho_{\mathrm{r}} c_{\mathrm{p}}} \\ + \frac{\tau_0}{\alpha_0 T_{\mathrm{c}} c_{\mathrm{p}}} \frac{\mathrm{D}}{\mathrm{D}t}\{\ell_1 \Gamma_1 + \ell_2 \Gamma_2\}.\end{aligned} \tag{2.3}$$

This system is closed with an equation of state that may be adequately approximated for the mantle application as

$$\rho = \rho_{\mathrm{r}}\left\{1 - \alpha_0 T_{\mathrm{c}} \alpha (T - T_{\mathrm{r}}) + \frac{1}{K_{\mathrm{T}}}(p - p_{\mathrm{r}})\right\} + \Delta_1(\Gamma_1 - \Gamma_{\mathrm{r}1}) + P_{\mathrm{r}} \Delta_2 (\Gamma_2 - \Gamma_{\mathrm{r}2}). \tag{2.4}$$

As written, this model system incorporates the influence of radial variations in all of the thermodynamic parameters, including thermodynamic phase. The latter variations are described in terms of the (dimensional) phase density functions

$$\Gamma_i = \tfrac{1}{2}\{1 + \tanh[(r_{\mathrm{p}i}(\boldsymbol{x}, t) - r) h_i / d]\}, \tag{2.5}$$

in which the radial position of the ith phase boundary $r_{\mathrm{p}i}$ is carried as a prognostic variable in the model, h_i is the thickness of the divariant phase-loop of the ith phase and d is the depth of the mantle, the length scale employed to develop the non-dimensional system. Other phase change related parameters appearing in the above system include the latent heats of reaction ℓ_1 and ℓ_2 and the density differences between olivine and spinel $\Delta_1 = \rho_2 - \rho_1$ and between spinel and the mixed perovskite–magnesiowüstite phase $\Delta_2 = \rho_3 - \rho_2$.

The principle non-dimensional groups in the above system include the Rayleigh number

$$Ra = \frac{\alpha_0 T_{\mathrm{c}} g_0 d^3}{\kappa_0 \nu_0}, \tag{2.6}$$

the dissipation number (see, for example, Peltier 1972), which multiplies the dissipation function Ψ in (2.3), namely

$$\tau_0 = \frac{g_0 \alpha_0 d}{C_\mathrm{p}}, \quad \tau(r) = \tau_0 \frac{g\alpha}{C_\mathrm{p}}, \tag{2.7}$$

and the non-dimensional heating per unit mass (in which ξ is the dimensional heating per unit mass)

$$\mu = \frac{\rho_0 \xi d^2}{k_0 T_\mathrm{c}}. \tag{2.8}$$

The subscript zero on an individual parameter in this system denotes a reference value and the symbols ρ, g, α, T, k, C_p, K_T, respectively, denote density, gravitational acceleration, coefficient of thermal expansion, temperature, thermal conductivity, specific heat capacity at constant pressure and isothermal compressibility. Further detailed discussion of the non-dimensionalization procedure can be found in Solheim & Peltier (1990, 1994*a*).

It is important to note that there are significant differences between the model of thermal convection embodied in (2.1)–(2.4) and the models that have been recently employed by others (see, for example, Zhao *et al.* 1992; Tackley *et al.* 1993) for the purpose of analysing phase-transition effects. Foremost among these differences is that the above model requires calculation of the complete history of phase boundary deflection at each interface in order to properly represent the physical effects induced by their presence rather than approximating the phase-transition effects as in Christiansen & Yuen (1984, 1985) by introducing local modifications of α and C_p. Assuming that the individual Clapeyron slopes, β_i say, may be taken to be independent of the thermodynamic coordinates, then we may integrate the Clapeyron equation to write

$$p = p_0 + \beta_i (T + T_0), \tag{2.9}$$

where, as in the above, T_0 is the (scaled by T_c) surface temperature. The β_i and ℓ_i are of course not independent but rather are related by the Clapeyron equation $\beta_i = \rho^2 \ell_i / (T_1 + T_0)\Delta_i$. If we define a transformed temperature A as

$$A = T - \frac{\tau_0}{\alpha_0 T_\mathrm{c} C_\mathrm{p}} (\ell_1 \Gamma_1 + \ell_2 \Gamma_2), \tag{2.10}$$

which can be usefully employed to replace the temperature T as prognostic variable in (2.3), then (2.9) generates the following two simultaneous equations:

$$\beta_1 \left\{ A + T_0 + \frac{\tau_0}{2\alpha_0 T_\mathrm{c} C_\mathrm{p}} [\ell_1 + \ell_2 + \ell_2 \tanh(r_{\mathrm{p}2} - r_{\mathrm{p}1}) h_2] \right\} + p_0 - p_\mathrm{r} = 0, \tag{2.11 a}$$

$$\beta_2 \left\{ A + T_0 + \frac{\tau_0}{2\alpha_0 T_\mathrm{c} C_\mathrm{p}} [\ell_1 + \ell_2 + \ell_1 \tanh(r_{\mathrm{p}2} - r_{\mathrm{p}1}) h_1] \right\} + p_0 - p_\mathrm{r} = 0, \tag{2.11 b}$$

which apply, respectively, at the positions of phase boundaries 1 and 2. At each azimuthal position in the spherical axisymmetric geometry that will be assumed to constitute the model domain, and at every time step in the numerical integration of the model equations, $r_{\mathrm{p}1}$ and $r_{\mathrm{p}2}$ are determined as the zeros of system (2.11). We expect that this detailed analysis will be required to preserve accuracy in the limit of high Rayleigh number when the phase boundary deflections $r_{\mathrm{p}1}$ and $r_{\mathrm{p}2}$ become large.

The physical influence on the convective circulation of the presence of a phase transition is exerted through two distinct effects, as first discussed from the perspective of linear stability theory by Busse & Schubert (1970) and employed in several early analyses of mantle dynamics, including those by Schubert *et al.* (1975) and Peltier (1973). These two effects are associated, respectively, with the influence on buoyancy of latent heat release (absorption) and with that which arises through phase boundary deflection because of the density difference between the phases. Whether the phase transition is exothermic or endothermic, these effects are opposite in sign insofar as their influence on radial mixing is concerned. Analysis presented in Peltier (1985) demonstrates that, in a spherical shell of mantle thickness and at the critical Rayleigh number for the onset of convection, the influence of latent heat release is dominant and stabilizing (destabilizing) for the exothermic (endothermic) transition. In this small amplitude limit the endothermic spinel–post-spinel transition will therefore be enhancing of convection, not inhibiting, so that it is only as a consequence of finite amplitude effects that there is any possibility that this boundary could promote the development of radial layering of the flow. As we shall see in the following section, the importance of phase boundary deflection rises so rapidly with Rayleigh number that for Earth-like values of this parameter (near 10^7) it overwhelms the influence of latent heat release, allowing the endothermic horizon to cause significant layering of the flow to develop.

3. Phase-transition-induced layered convection

In the recent papers by Peltier & Solheim (1992*a, b*) and Solheim & Peltier (1994*a, b*), the model embodied in the field equations (2.1)–(2.5) has been employed to perform a rather exhaustive exploration of the dependence of the nature of the convective circulation upon the model parameters when the Rayleigh number is fixed to the Earth-like value of 10^7. Although these integrations have been performed in axially symmetric spherical geometry, in order that a realistic Rayleigh number could be employed, the parameter dependencies revealed by these analyses have recently been confirmed in detail at lower Rayleigh number in a three-dimensional spherical model by Tackley *et al.* (1994). In this section I will describe only the main result of this work concerning the central issue of layered convection. The interested reader is referred to the references cited in the bibliography for additional detail.

In figure 2 I show time series for three characterizations of the convective circulation delivered by the model when the shell is assumed to be heated entirely from below ($\mu = 0$) and when the Clapeyron slopes for the olivine–spinel and spinel–post-spinel transitions are taken to be $\beta_1 = +3\,\mathrm{MPa\,K^{-1}}$ and $\beta_2 = -2.8\,\mathrm{MPa\,K^{-1}}$, respectively. As demonstrated in Solheim & Peltier (1994*a*), the choice $\mu = 0$ minimizes the tendency to layered flow. Similarly, the choice for β_1, which is on the upper bound of the experimental measurements, also minimizes the ability of the deeper transition to cause layering since the influence of the shallower exothermic transition is to enhance radial mixing. Furthermore, the magnitude selected for β_2 is well within the range suggested by the experimental measurements of Ito & Takahashi (1989) and Ito *et al.* (1990). Inspection of the time series in figure 2 for the mean temperature of the shell $\langle T\rangle$, the mean tangential velocity at the upper surface of the shell $\langle u_{\mathrm{surf}}\rangle$ and the radial mass flux at the depths of the 400 and 660 km phase transitions, respectively, demonstrates that the flow exhibits a high degree of temporal 'intermittency'. Intense pulses of high activity separated in dimensional

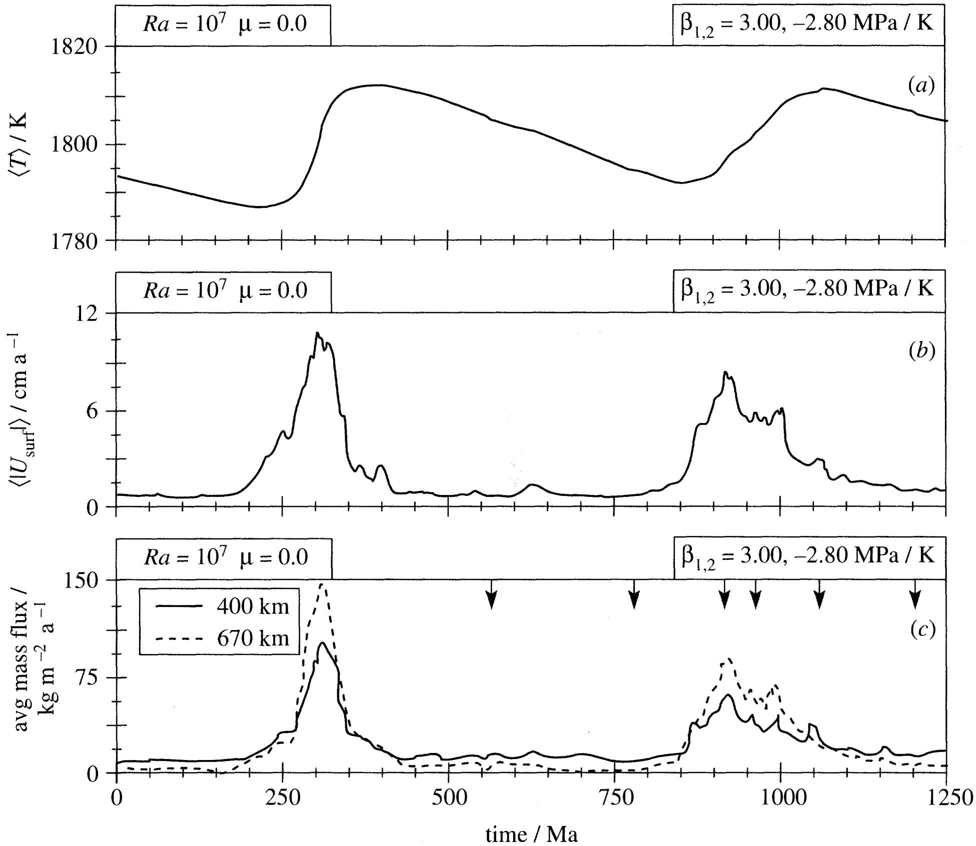

Figure 2. Time series from a multiphase convection simulation for a model heated entirely from below ($\mu = 0$). $\langle T\rangle$ is the mean temperature of the mantle while $\langle U_{\mathrm{surf}}\rangle$ is the average tangential speed of material motion at the surface. Part (c) shows time series of the azimuthally averaged mass flux across the olivine–spinel and the spinel–post-spinel phase boundaries. See the text for discussion.

time by approximately 600 Ma develop episodically, the intervening episodes being characterized by relative quiescence and much reduced mass flux across the 660 km horizon.

The nature of the spatial reorganization of the convective circulation that accompanies the temporally episodic bursts of mixing is documented in figure 3, on which are shown six individual depictions of the mantle temperature field corresponding to the six times denoted by the arrows on figure 2c. Inspection of this sequence of images demonstrates that the regime of vigorous mixing develops in response to the occurrence of intense 'avalanches' of cold material from the transition zone into the lower mantle. Before the occurrence of the avalanche events documented in figure 3 (these downwellings are clearly observed at 910 and 960 Ma) the flow is in a highly layered state, a state to which it returns once an individual sequence of avalanches has subsided. The time-dependent extent of the layering is characterized quantitatively in figure 4a on which is shown the radial mass flux diagnostic F_{m}, first introduced by Peltier & Solheim (1992a), which is defined as

$$F_{\mathrm{m}}(r,t) = \langle \rho_{\mathrm{r}}|u_{\mathrm{r}}|\rangle \Big/ \frac{1}{(r_{\mathrm{B}} - r_{\mathrm{cmb}})} \int_{r_{\mathrm{cmb}}}^{r_{\mathrm{s}}} \langle \rho_{\mathrm{r}}|u_{\mathrm{r}}|\rangle \, \mathrm{d}r. \tag{3.1}$$

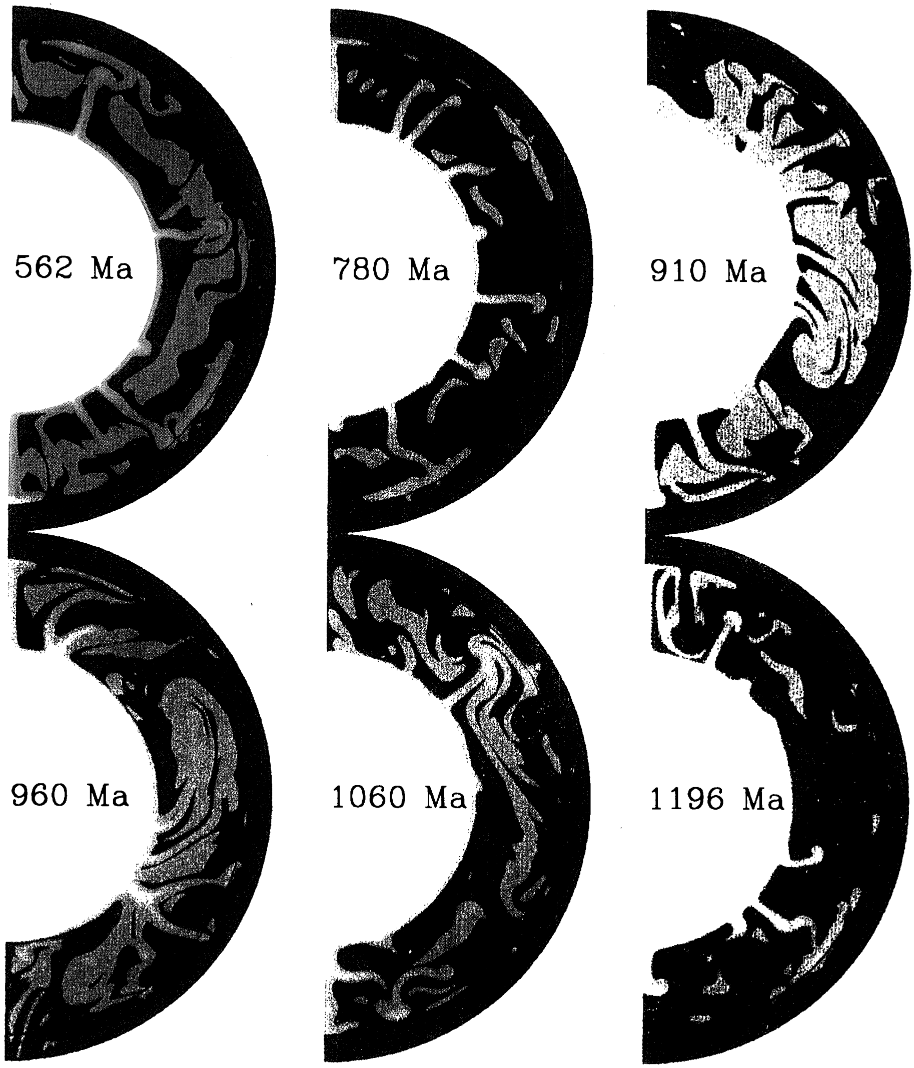

Figure 3. Temperature field within the axisymmetric mantle shell at the six instants of time marked by the arrows in figure 2*c*. Note the intense cold (blue) 'avalanches' at 910 and 960 Ma that cross the 670 (660) km discontinuity into the lower mantle. At other times the flow is very strongly layered.

In figure 4*a*, I show temporally averaged depth dependent F_m for four different epochs within the time series shown in figure 2, two of which are periods during which the flow is clearly layered (F_m small at the depth of the endothermic horizon) and two of which correspond to periods when intense avalanches are occurring. It is very clear by inspection of this figure that the extent to which mixing is suppressed across the 660 km horizon during the extensive episodes of layering is extreme.

In order to understand the physical mechanism that is responsible for supporting the spatial and temporal intermittency that is characteristic of the circulation in this phase-transition modulated regime, one need only focus upon the geotherm that is characteristic of these flows. In figure 4*b* are shown temporally averaged geotherms obtained from long integrations for heated from below flows at $Ra = 10^7$ that differ

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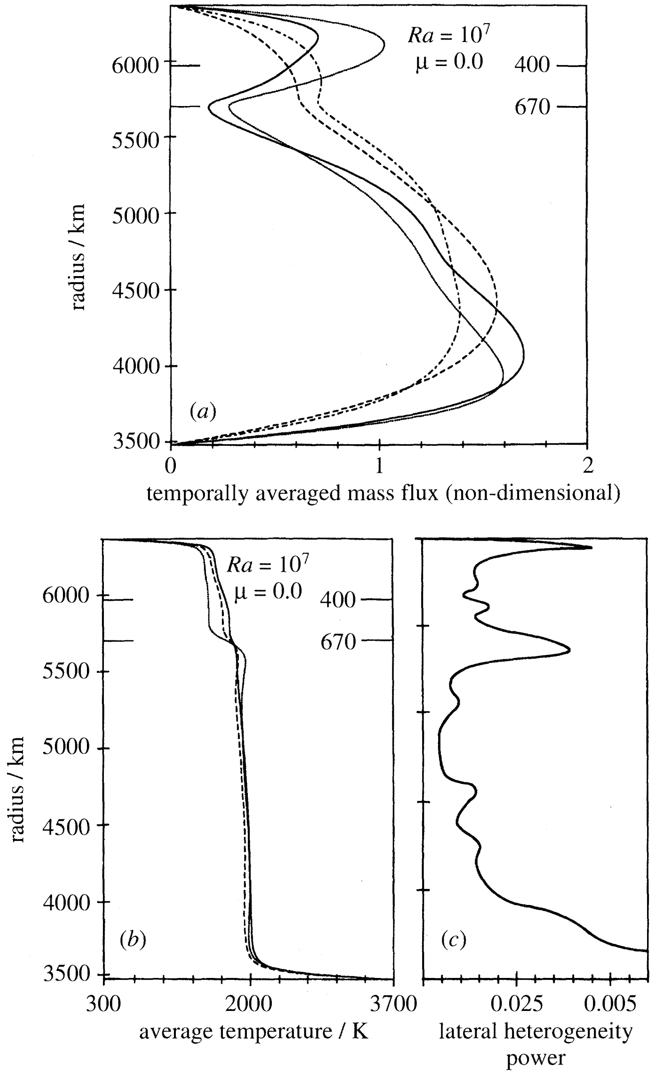

Figure 4. (a) Azimuthally and temporally averaged mass flux across the 670 (660) km seismic discontinuity from four intervals of time for the simulation from which temperature fields are shown on figure 3. (b) Azimuthally and temporally averaged geotherms from three heated from below convection simulations which differ only in the value assumed for the Clapeyron slope of the 670 (660) km phase transformation. The solid line, dashed line and dotted line are geotherms for $\beta_2 = -2$, -2.8 and -4 MPa K^{-1}, respectively. (c) Depth dependent lateral heterogeneity power spectrum (time averaged) from the heated from below convection model at $Ra = 10^7$. Note the sharp peak in this spectrum that is predicted to be coincident with the spinel–post-spinel phase transformation.

from one another only in the value of the Clapeyron slope β_2 that is taken to characterize the endothermic transition at 660 km depth. Results are shown for values of $\beta_2 = -2.0$, -2.8 and -4.0 MPa K^{-1}. Inspection of the figure demonstrates that as the magnitude of the Clapeyron slope of the endothermic transition increases through this range, a progressively more intense thermal boundary layer develops across the phase boundary. As discussed in detail in Solheim & Peltier (1994a), the spatial and

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temporal intermittency in these flows may be simply understood through analysis of the local stability of this internal thermal boundary layer. This fact is made clear in figure 5, on which is shown the time series for the radial mass flux at 660 km depth (figure 5*a*, which is for the same simulation discussed previously in connection with figure 3), the time series for the azimuthally averaged temperature difference across the thermal boundary layer at this depth (figure 5*b*), and the time series for the azimuthally averaged boundary layer thickness (figure 5*c*). Using the latter two data, one may calculate a boundary layer Rayleigh number $Ra^{\delta}(t) = g_0\alpha\Delta T_{\delta}\delta^3/\kappa\nu$, in which the values for the parameters, g, α, κ and ν are taken to be equal to their values at the position of the endothermic horizon. The time series $Ra^{\delta}(t)$ for this example simulation is shown in figure 5*d*, inspection of which demonstrates that $Ra^{\delta}(t)$ reaches a maximum value near 600 *before* the onset of each of the avalanche events in the sequence. During the avalanche Ra^{δ} drops to zero as is expected since the avalanche destroys the thermal boundary layer through whose convective instability it was engendered. A critical value of Ra^{δ} near 10^3 might be naively expected on the basis of a local linear stability argument (see, for example, Chandrasekhar 1960), but in fact the control of the intermittency by a condition of this kind can be understood only on the basis of a detailed local analysis of boundary layer stability that includes both the influence of the endothermic phase transition and the convergent motion onto the phase boundary itself (Peltier *et al.* 1996). The spatial and temporal intermittency of the mantle convective circulation in the phase-transition modulated regime may therefore be understood as being driven by the episodic creation and destruction through convective instability of the thermal boundary layer that straddles the endothermic horizon.

In comparing the results of the analyses described here to those obtained in the earlier literature, I view three such prior analyses of the finite amplitude problem to be especially worthy of comment. These are, respectively, the work by Richter (1973), Richter & McKenzie (1981) and Christiansen & Yuen (1984, 1985). To various degrees, all of these early analyses either dealt with, or at least acknowledged the possible importance of, the issue of the impact of a phase transformation upon convective mixing. The paper by Richter (1973) is important as it was apparently the first to properly formulate the finite amplitude dynamical problem, although here one might reasonably question the assumption that a number of non-Boussinesq effects could be ignored. As the analyses presented in this paper were restricted to the regime of small supercriticality and dealt solely with the impact of the exothermic olivine–spinel transition on the flow, they have not substantially influenced more recent research. The paper by Richter & McKenzie (1981) focused upon the idea of layered convection arising in consequence of a chemical density contrast across the 660 km horizon and argued that this rigidly layered style of convective mixing was strongly suggested by the accumulating evidence from trace element geochemistry (see, for example, O'Nions *et al.* 1979; Wasserburg & De Paolo 1979). Although not supported by any direct analysis, this paper also contains the following prescient comment concerning the possible importance of an endothermic phase transformation that 'it may be that a phase change with negative Clapeyron slope may itself be capable of acting as a barrier to vertical motions under conditions appropriate to the mantle'. At this time it was not known on the basis of direct experimental measurement what the Clapeyron slope of the spinel–post-spinel transition was, or indeed whether it was actually negative. The initial measurements of this critical quantity were not available until 1989 when Ito & Takahashi published their results

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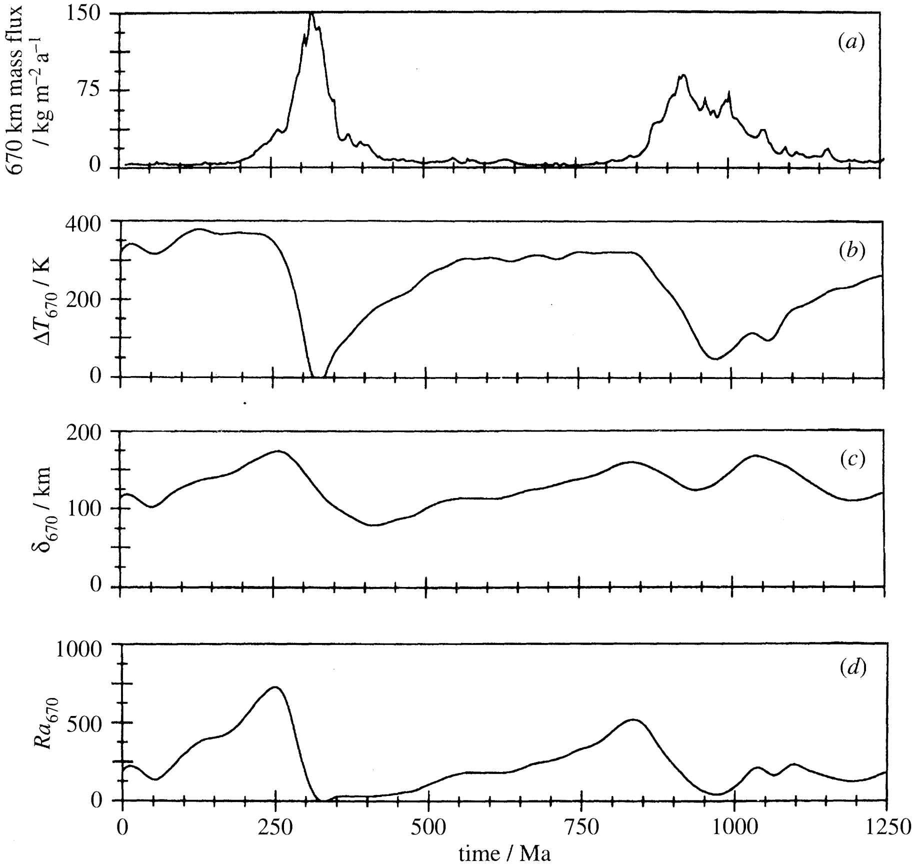

Figure 5. Four diagnostic time series from a heated-from-below convection model with multiple phase transitions. (*a*) Radial mass flux across 670 km depth; (*b*) temperature contrast across the 670 km thermal boundary layer; (*c*) thickness of the 670 km thermal boundary layer; (*c*) boundary layer Rayleigh number.

from large-volume high-pressure multi-anvil experiments. Insofar as the analyses of Christiansen & Yuen (1985) were concerned, their model calculations extended only to the rather low value of $Ra = 2 \times 10^6$. Although their work did validate the suggestion of Richter & McKenzie, by showing that a phase transition with sufficiently negative Clapeyron slope could induce a layered style of flow in a constant viscosity model, their computed value of the Clapeyron slope that would be required to induce layering was rather high and was obtained only by extrapolation from numerical results obtained at Rayleigh numbers considerably lower than that which corresponds to the present day Earth. Specifically, they state (Christiansen & Yuen 1985) that 'Scaling the model results to the 670 km discontinuity in the Earth's mantle as a possible endothermic phase boundary, we estimate the critical Clapeyron slope to be in the range -4 to -8 MPa K^{-1} (-40 to -80 bars K^{-1})'. In the decade that has passed since these early analyses were performed we have learned not only that the spinel–post-spinel transition *is* endothermic but also obtained reasonably accurate experimental estimates of its Clapeyron slope.

The most recent flurry of research activity, represented by the papers of Machetel & Weber (1991), Peltier & Solheim (1992*a*, *b*), Solheim & Peltier (1993, 1994*a*, *b*), and Tackley *et al.* (1993, 1994), was essentially triggered by the high-pressure experimental measurements of β_2 by Ito & Takahashi (1989) and Ito *et al.* (1990). In Ito & Takahashi (1989) the experimentally determined value of β_2 was stated to be between -2 and -6 MPa K^{-1} or -4 ± 2 MPa K^{-1} (the recent result of Chopelas *et al.* (1994) is -4.0 ± 0.4 MPa K^{-1}). Given that the best estimate for the critical value of the Clapeyron slope for the transition into the layered state according to Christiansen & Yuen (1984, 1985) was near -6 MPa K^{-1}, according to these authors the capacity of the endothermic transition to induce a layered style of flow would be marginal at best.

The notion that the effect might nevertheless be important was suggested in the first of the most recent series of analyses, namely in the axisymmetric spherical simulations of Machetel & Weber (1991) whose results were obtained for a maximum value of the Rayleigh number of 2×10^6, essentially the same value as had been reached in the analyses of Christiansen & Yuen (1985). At this value of the Rayleigh number, Machetel & Weber (1991) found that significant layering seemed to be characteristic of the flow for $\beta_2 = -4$ MPa K^{-1} although the degree of layering was not quantified. For $\beta_2 = -2$ MPa K^{-1} no significant effect was discernable at this Rayleigh number. The analyses presented in Peltier & Solheim (1992), in contrast, were performed in the same axisymmetric spherical geometry as that employed by Machetel & Weber (1991), but for a Rayleigh number $Ra = 10^7$ appropriate to that of the present day Earth. It was therein demonstrated that with $\beta_2 = -2.8$ MPa K^{-1}, well within the bounds originally suggested by Ito & Takahashi (1989) and Ito *et al.* (1990), the flow was *strongly* layered, a state that appeared to be episodically disrupted on a timescale near 600 Ma. In Solheim & Peltier (1994), cited before its appearance by Tackley *et al.* (1993), these episodic disruptions of the layered state were first likened to 'avalanches' and shown to arise in the manner previously discussed, through the convective destabilization of the thermal boundary layer that develops across the endothermic phase transition when the Rayleigh number is 'Earth-like" and the Clapeyron slope β_2 is as measured experimentally (appropriately diluted to account for the fractional density of olivine). In Peltier & Solheim (1992) it was also shown by direct calculation that the convection-enhancing effect of the exothermic olivine–spinel transition was not sufficient to influence in any significant way the ability of the deeper endothermic horizon to enforce a predominantly layered state of flow. Based upon the high Rayleigh number results of Solheim & Peltier (1994*a*, *b*), it is now entirely plausible therefore that the convective circulation in the modern Earth could be significantly layered, due solely to the influence of the endothermic phase transition. Since the degree of layering increases with the Rayleigh number (Christiansen & Yuen 1985; Solheim & Peltier 1993, 1994*a*) it also seems clear that the circulation would have been more strongly layered (the layering less often disrupted by the 'avalanche effect') earlier in Earth history. This is assuming that the constant viscosity analysis, which also neglects the influence of the surface plates, will not prejudice the results.

4. Implications of the episodically layered mantle convection scenario

If the dynamical regime of Earth's mantle is well described by the layered intermittency scenario described above then there are clearly a number of implications that deserve to be explored. For example, one consequence of the episodic destruction of the layering is that the rate of heat loss from the core is expected to be highly time dependent. This is illustrated in figure 6a, in which a series of instantaneous profiles are shown for the radial heat advection from a simulation in which a typical sequence of avalanches develops. The model for which results are shown here has $\beta_1 = 3\,\mathrm{MPa\,K^{-1}}$, $\beta_2 = -2.8\,\mathrm{MPa\,K^{-1}}$ and μ corresponding to 50% heating from within and 50% heating from below. Given the large amplitude of the variability (from 20 to 100 TW with the highest heat flow in this sequence of profiles obtained at the time of an avalanche event; the solid line indicates the average of the five instantaneous profiles shown) we must expect that the impact upon the dynamics of the core would be significant, sufficient perhaps to play an important role in the process of polarity reversal of the geomagnetic field. It also seems clear that this scenario has much to recommend it from the perspective that it would appear to allow a rather straightforward interpretation of the constraints on mantle mixing that are provided by both the previously cited data of trace element geochemistry and more recent results (see, for example, O'Nions & Tolstikhin 1994). As discussed at length in Peltier *et al.* (1996), this scenario also has potential for helping to understand the details of the so-called Wilson cycle of supercontinent agglomeration and breakup. My main purpose in this section, however, is to point out an additional conundrum that may be resolved by this new paradigm. This concerns the question of the magnitude of the momentum diffusivity that characterizes the mantle and its variation with depth.

The present circumstance with regard to the inference of this important transport coefficient is contentious. On the one hand, both detailed formal inversions of the data of postglacial rebound (Mitrovica & Peltier 1993, 1995; Peltier & Jiang 1996a, b) and results from the solution of associated forward problems (Peltier 1974, 1976, 1985; Peltier *et al.* 1986, 1990) strongly constrain the viscosity to remain rather uniform to a depth near 1400 km and close in average value to that originally inferred by Haskell (1937), namely 10^{21} Pa s (the average value in the upper mantle and transition zone may be as much as three times lower than this and the value at the top of the lower mantle marginally higher). On the other hand, interpretations of the non-hydrostatic geoid of the planet in terms of convection models, the internal density structure of which is constrained by seismic tomography, have generally led their authors to conclude that the viscosity of the mantle beneath 660 km depth was dramatically higher than that of the overlying region, the often preferred ratio being 30 at *minimum* (see, for example, Richards & Hager 1984; Hager & Richards 1990). Although this generally preferred value has been shown to be unnecessarily extreme (Forte & Peltier 1987, 1991, 1994; Pari & Peltier 1995), there nevertheless continues to exist a significant disparity between the results obtained on the basis of these two distinct methods of inference. As discussed in Peltier (1985) and Peltier *et al.* (1986), one way in which one might reconcile this conflict is to invoke the notion of transient rheology and to argue that on the long timescale associated with the thermal convection process the lower mantle is governed by a higher effective viscosity than that which controls the dynamics on the significantly shorter timescale of postglacial rebound. Before advocating such an interpretation, however, it is clearly important to understand whether or not it might be required by the data. As it happens, the structure of the depth-dependent azimuthally averaged temperature field in phase-transition modulated thermal convection, illustrated previously in figure 4b, suggests an alternative possibility. This alternative will be explored in the remainder of this section.

As discussed in Peltier (1985), the approximate uniformity with depth of the vis-

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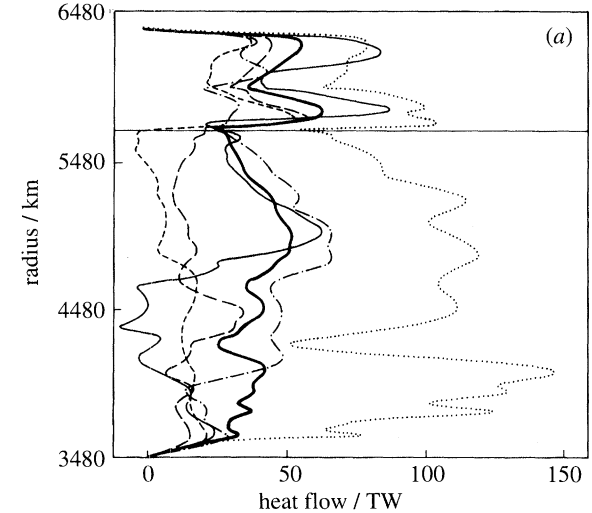

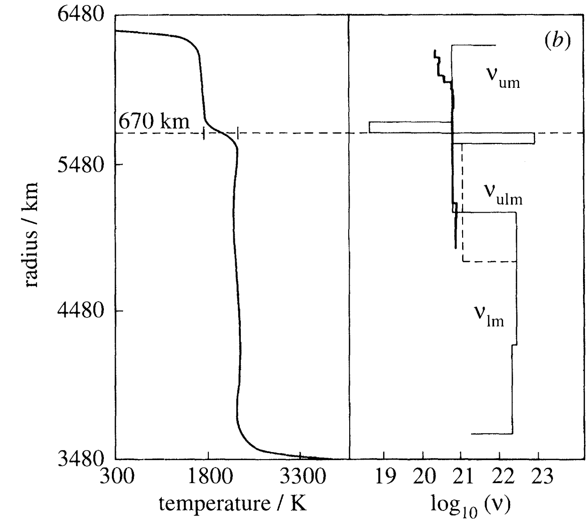

Figure 6. (*a*) A sequence of depth-dependent azimuthally averaged advected heat flow profiles from a convection model operating at Rayleigh number of $Ra = 10^7$ and having 50% heating from within and 50% heating from below. The heavy black curve is the average of the five instantaneous curves shown, the dotted member of this sequence in which the heat flow is typically double the average corresponds to a time during which an avalanche is occurring, whereas the other four are taken at times when the flow is layered. The radial average of the heavy black curve gives a heat flow that is very close to the 34 TW subcrustal contribution characteristic of the modern Earth. (*b*) A typical geotherm from a flow that is significantly layered by the influence of the endothermic transition (left). A sequence of viscosity profiles for the mantle (right). The thin solid line is the profile referred to as ν_b in the text, whereas the heavy black curve is the constraint on viscosity recently delivered by formal inversion of the realtive sea level data of postglacial rebound (Mitrovica & Peltier 1995). The dashed curve represents a variant on model ν_b which illustrates the increased depth to which the ultimate increase of viscosity in the lower most mantle may be pushed by slightly increasing the viscosity contrast across the 660 km interface itself.

cosity of the mantle averaged over a length scale of 300–600 km (to a depth of order 1400 km) that is required by the data of postglacial rebound, when coupled with a temperature profile with a strong internal thermal boundary layer such as that shown in figure 4*b* for the model with $\beta_2 = -4\,\mathrm{MPa\,K^{-1}}$, may be taken to suggest the existence of a dipolar viscosity variation across the 660 km phase-transition interface. It is expected that this variation would be characterized by a thin layer with low viscosity above the transition and perhaps an equally thin layer with high viscosity below the transition, as sketched on figure 6*b*. Since direct computation of the signatures of postglacial rebound for models that contain a structure of this kind (not shown) demonstrate that it would be essentially undetectable when mean upper mantle and transition zone and upper part of the lower mantle viscosities are taken to be near 10^{21} Pa s, an interesting issue then concerns the question as to whether a model of this kind is capable of explaining the structure of the non-hydrostatic geoid in the context of a convection model based upon the use of seismic tomography to constrain the density field.

This issue may be addressed most transparently by simply employed the anelastically compressible three-dimensional spherical convection model of Forte & Peltier (1991) to directly compute the non-hydrostatic geoid. This model is based upon the following set of field equations:

$$\nabla \cdot \boldsymbol{u} = -(\dot{\rho}_\mathrm{r}/\rho_\mathrm{r})u_\mathrm{r}, \tag{4.1}$$

$$\sigma_{ij,j} + \rho_\mathrm{r}\psi'_i - \rho' g_0 \hat{r} = 0, \tag{4.2}$$

$$\sigma_{ij} = -p'\delta_{ij} + \eta(u_{i,j} + u_{j,i} - \tfrac{2}{3}u_{k,k}\delta_{ij}), \tag{4.3}$$

$$\psi'_{,kk} = -4\pi G\rho', \tag{4.4}$$

in which it is assumed that the molecular viscosity $\eta = \rho\nu$ is a function of radius only, $\dot{\rho}_\mathrm{r} = \mathrm{d}\rho_\mathrm{r}/\mathrm{d}r$, ψ' is the perturbation of the gravitational potential, G the universal gravitational constant and σ_{ij} the deviatoric stress tensor. The system (4.1)–(4.4) may be solved, given prescribed density heterogeneity $\rho'(\boldsymbol{r})$, using the Green function method of Forte & Peltier (1991) with $\rho'(\boldsymbol{r})$ assumed to be adequately represented by a model derivative of the application of global seismic tomography. The results that I will present here will be based entirely upon the model SH8WM13 of Woodward *et al.* (1993), a model which represents the lateral variations of s-wave velocity in terms of a spherical harmonic expansion truncated to degree and order eight and the radial variations using an expansion in Chebychev polynomials truncated at order 13. In order to obtain the density field $\rho'(\boldsymbol{r})$ that is required to drive the convective circulation according to (4.1)–(4.4), I will employ the same simple functional form for the depth-dependent conversion factor $\partial \ln v_\mathrm{s}/\partial \ln \rho$ that was used in Pari & Peltier (1995), which equals 0.22 from the surface to a depth of 670 km, below which it decreases linearly to a marginally lower value of 0.15 at the core–mantle boundary.

In figure 7 I compare theoretically predicted geoids based upon the model embodied in (4.1)–(4.4) with the GEM-T2 geoid (Marsh *et al.* 1992) in terms of fields synthesized from coefficients complete to degree and order eight. Theoretical predictions are shown for three viscosity models, respectively, labelled ν_a, ν_b and ν_c. With the degree two and order zero coefficient set to equal the observed coefficient, these theoretical models all reduce approximately 80% of the variance in the observed field through this range of wavenumbers. Model b, which is illustrated by the thin solid line on figure 6*b*, contains the above discussed dipolar structure across the 660 km phase-transition interface and is marginally the best fitting, in a variance reduction sense, of the three models shown. Model a is that previously suggested by

Forte *et al.* (1992) and one which contains the soft component of the dipole above the interface but not the stiff component below. Because it lacks the latter feature, the model requires a jump in viscosity to a value near 4×10^{21} Pa s below 660 km depth (if the average upper mantle and transition zone value is fixed to 10^{21} Pa s) and a steady increase to a lower mantle value 20 times in excess of this by a depth 200 km above the CMB. This model is not ruled out by postglacial rebound data if the viscosity is a simply reduced by a factor of three everywhere, this being possible because the geoid datum is sensitive only to viscosity contrast rather than to the absolute value of viscosity. Furthermore, the reduction of the average viscosity in the upper mantle and transition zone to a value of 0.3×10^{21} Pa s has been shown to be required on the basis of formal inversions of several different data sets pertaining to the post-glacial rebound process (Peltier & Jiang 1996*a*, *b*). Model ν_c differs from a in that the viscosity immediately below 660 km depth is seven times higher than the upper mantle value and remains so to a depth of 1400 km, below which it increases further and discontinuously to the same maximum lower mantle value as in a. The point of interest here is that all of these models provide satisfactory accord with the data although they may differ somewhat in their ability to satisfy the constraints on the viscosity of the mantle that have recently been derived on the basis of formal inversion of the relative sea-level data of postglacial rebound (Mitrovica & Peltier 1993, 1995; Peltier & Jiang 1996*a*, *b*). An example of a rebound derived viscosity profile, which shows minimal variation of viscosity from the upper mantle to the lower mantle, is shown on figure 6*b* as the thick solid line that extends from the surface to a depth near 1400 km, below which the relative sea-level data employed in the inversion have no resolving power. The model represented by the dashed line in figure 6*b* illustrates the amount by which the viscosity beneath 660 km depth must be increased in order to increase the depth below which the lower mantle viscosity is further enhanced from 1200 to 1400 km, this latter feature being required by the non-hydrostatic geoid data. Detailed analyses of these models and further discussion of their implications can be found in Pari & Peltier (1995). The main insight to be gathered from these analyses, however, is that to the extent that post-glacial rebound data may be invoked to argue that the upper mantle–lower mantle viscosity contrast is weak then non-hydrostatic geoid anomalies appear to be best fit by models with quite sharply localized variations of viscosity near 660 km depth, the low viscosity layer immediately overlying this horizon being most strongly constrained.

As commented in the abstract, there remains a most interesting further pair of constraints that may be brought to bear upon the viscosity profile of the deep Earth and which does allow us to more fully verify the plausibility of profiles of the above described type, especially concerning the elevation of viscosity in the deepest part of the lower mantle. These constraints derive from observations of both the non-tidal acceleration of planetary rotation and the present day wander of the rotation pole towards Greenland at a rate near 0.95° per 10^6 years that is evident in the pole path deduced on the basis of the data of the International Latitude Service. Recent reanalyses of these data (Peltier & Jiang 1994, 1996) have confirmed earlier suggestions to the effect that both observations are most straightforwardly interpreted as being due to the continuing process of glacial isostatic adjustment (Peltier 1982; Wu & Peltier 1984). These analyses have furthermore shown that when the mantle viscosity profile is fixed to ν_a or ν_b by the relative sea-level data of postglacial rebound in the depth range less than 1400 km, then the viscosity of the deepest part of the lower mantle

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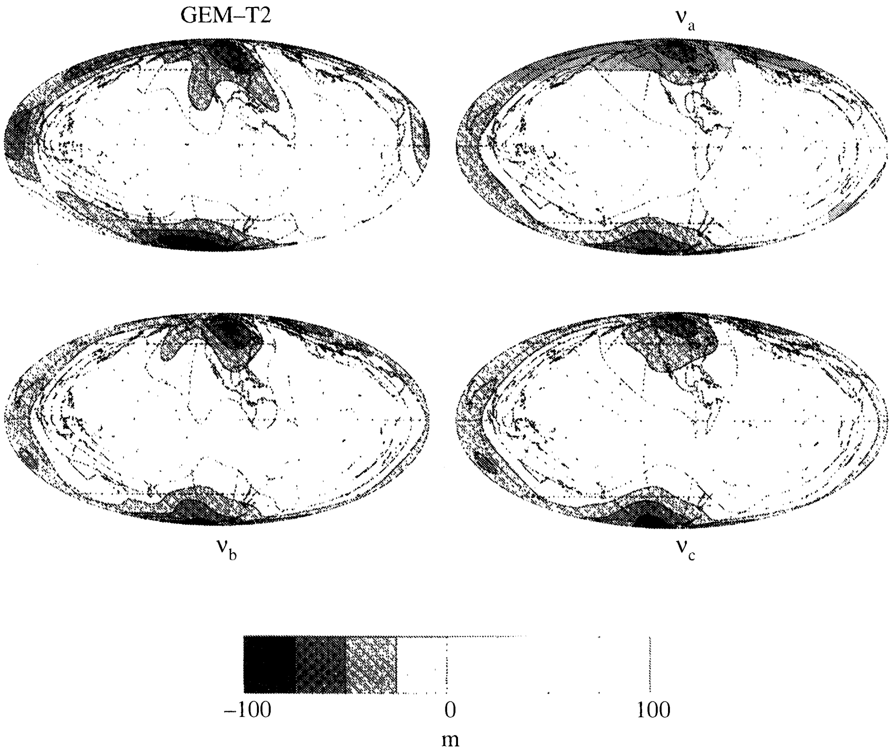

Figure 7. Observed non-hydrostatic geoid in the degree range $2 \leqslant \ell \leqslant 8$ (GEM–T2) and predicted non-hydrostatic geoids for the three viscosity models ν_a, ν_b and ν_c described in the text.

must be near the value of 10^{22} Pa s. The latter is entirely compatible with the value required by the non-hydrostatic geoid data discussed herein.

Comparison of the analyses presented in §§ 3 and 4 of this paper will demonstrate that we are not yet in a position to claim to have produced a fully self-consistent dynamical description of the implications of the layered convection scenario for the variation of the momentum diffusivity. The development of such a description will clearly require the coupling of the phase-transition effects upon which I have focused here with the effects that are expected to arise due to the temperature, pressure and perhaps phase dependence of viscosity. Efforts in this direction are the basis of ongoing research and will be reported elsewhere.

5. Discussion and conclusions

The discovery of the profound effect that the endothermic phase transformation at 660 km depth could have upon mantle mixing has substantially transformed the continuing debate on the issue as to whether the mantle convective circulation is 'layered' or 'whole-mantle' in style. According to the results of calculations performed at the Earth-like Rayleigh number of 10^7, the critical value of the Clapeyron slope beyond which significant layering results is less than -3 MPa K^{-1}, and therefore a factor of two lower than that suggested by Christiansen & Yuen (1985). Since recent experimental measurements of β_2 have constrained its value to be in this range, it would appear that significant layering is probable even if, as also remains plausible, there were no increase of the mean atomic weight of mantle material across the 660 km horizon. In this regard it should also be clear that if strong phase-transition-induced layering were to have existed in the early Earth, this could have led to the development of a chemical discontinuity at 660 km depth and therefore to a marked increase in the degree of layering. An uncertainty continues to exist, however, concerning the impact of the garnet component of mantle mineralogy upon buoyancy effects across the 660 km horizon and both experimental and theoretical analyses of this impact are clearly required.

Of greatest interest in the immediate future will be efforts to confirm or to deny the existence of phase-transition-induced layering based upon pertinent observational constraints. Of the avenues of attack that are liable to prove most fruitful in this respect, one will surely continue to be global seismic tomography. In this context much of the ongoing effort has come to be focused upon the search in global seismic models for the peak in the power spectrum of the lateral heterogeneity that Peltier & Solheim (1992) predicted to develop on the endothermic horizon when the flow is layered. This feature is illustrated in figure 4*c* from a typical simulation with $Ra = 10^7$ and $\mu = 0$. The issue of the ability of global seismic models to resolve such a feature has been discussed recently in Peltier *et al.* (1995). One further line of investigation that may yield additional insight concerns analyses of the non-hydrostatic geoid, such as those briefly described in the last section. It is generally believed (see, for example, Hager 1984) that it is not possible to reconcile this constraint with a layered model of the convective circulation. If this were true it would strongly rule out the layered convection scenario unless one were to argue that the layering were intermittent (as suggested by the *a priori* analyses of the influence of phase transitions on convection discussed herein) and the flow at present were in an essentially whole mantle state.

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Discussion

R. K. O'Nions (*University of Oxford, UK*). The tendency of the endothermic phase change at 670 km to induce a layered mantle is entirely consistent with geochemical observations which require layering. In particular, the rare gases provide the most powerful constraints and suggest that layering has been in operation since *ca.* 4.3 Ga ago. Furthermore the rare gases suggest that the lower to upper mantle mass flux is about 1% of the lower mantle per Ga over the last 3–4 Ga. This appears to contradict

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the relatively frequent avalanches that Professor Peltier's model predicts. How robust is the conclusion concerning both the existence and frequency of such avalanches?

W. R. Peltier. Professor O'Nions's question is clearly a most important one if the scenario of phase transition induced layering is to provide a successful means of reconciling the seismological and rare gas isotopic data. Our analyses of the impact of the endothermic transition upon layering, under the assumption that the viscosity of the mantle may be assumed constant, demonstrate that for a Rayleigh number of 10^7 (as is approximately characteristic of the modern Earth) the circulation is very significantly layered although the layering is episodically disrupted by intense avalanche instabilities, during which significant mass exchange occurs between the lower mantle and the upper mantle and transition zone. In the past, when the mantle Rayleigh number was significantly higher, these avalanche events would have been less frequent and more short lived leading to a further reduction in the mixing. One should think of the degree of layering of the circulation in the constant viscosity flows as exhibiting a sigmoidal dependence upon the Rayleigh number. At $Ra \leqslant 10^6$ layering is weak, at $Ra \approx 10^7$ layering is strong but episodically disrupted by avalanches and at $Ra \approx 10^8$ layering is essentially perfect. Initial work in my own laboratory and in other groups, however, suggests that this phenomenology is quite strongly influenced by the pressure and temperature dependence of viscosity. Initial indications are that the variations of viscosity so induced cause the degree of layering to increase above that predicted by the uniform viscosity models and this speaks directly to Professor O'Nions's question concerning the robustness of the results that have been obtained to date. How these results will be further modified once the large horizontal scale coherence of the flow due to the surface plates is introduced into the model remains to be seen. In my view this latter issue is a most important one and until it is properly settled we have no reason to be entirely secure in our understanding of the importance of the endothermic phase transition effect. What we can say is that the results already in hand are strongly suggestive that this effect may fundamentally influence the evolution of the physical and chemical state of the planet.

J. H. Davies (*Department of Earth Sciences, University of Liverpool, UK*). The 'avalanche' process described allows the possibility of reconciling the observations of seismic tomography for significant upper mantle to lower mantle transfer at subduction zones, with the constraints for limited transfer from rare-gas isotope geochemistry. Though it is true that regional tomography shows clearly that some slabs do not penetrate immediately into the lower mantle but rather flatten above the interface; most importantly these same studies equally show other slabs which do not collect at the interface (Zhou & Clayton 1990; Fukao *et al.* 1992; van der Hilst *et al.* 1991; Ding & Grand 1994; Grand 1994). In many of these latter cases high velocity anomalies have been found in the lower mantle. In these cases I agree with Professor Grand that the simplest (though not required) explanation is that the subducting slabs in these places have passed through into the lower mantle (Ding & Grand 1994). Another possibility is that these velocity anomalies follow a major downward deflection of a chemical boundary between upper and lower mantle; i.e. no transfer. This, at least in Professor Grand's work (1994), where he has imaged a slab-like seismic anomaly extending down nearly to the core mantle boundary, seems implausible. Of course seismology only gives us a snapshot of the present, while the rare gas isotope geochemistry commented on by Professor O'Nions (O'Nions & Tolstikhin 1994) provides an integrated constraint over geologic history. As you have

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pointed out, it is possible that the phase change could have been a greater barrier to penetration in the geologic past when the Rayleigh number of mantle convection was greater. Do you agree that your work provides a possible means of reconciling the seismic tomography evidence for the present day passage of material between the upper and lower mantle with the geochemistry constraint for limited exchange over geological time?

W. R. Peltier. Professor Davies's commentary raises two rather different issues, it seems to me. Firstly there is the issue of what should be implied on the basis of results from local seismic tomographic reconstrucitons of the lateral heterogeneity of seismic velocity in the vicinity of subduction zones. If the results from the high Rayleigh number convection simulations that I have reported may be mapped directly into this phenomenology, then the simulations suggest that slabs characterized by the largest temperature heterogeneity should be most strongly impeded in their descent by the 660 km endothermic transition. This material is predicted to first collect above the phase tranistion interface, leading to the steepening of the thermal boundary layer locally and thereby eventually to the occurrence of an instability of avalanche type. The same model suggests that slabs characterized by a weaker lateral heterogeneity will be relatively unimpeded in their descent if the 660 km interface is such that no change in mean atomic weight accompanies the endothermic transition. One therefore expects to see a range of degrees of connectivity (correlation) between the lateral heterogeneity above and below the interface. This would appear to be in accord with the seismological constraints. The second of the issues raised concerns the question as to how the degree of inhibition of radial mixing might have varied in the past. On this point the convection simulations are very clear in demonstrating that the degree of layering of the flow increases strongly with the Rayleigh number in the range $Ra \geqslant 10^6$. At $Ra = 10^7$ strong layering is episodically disrupted by avalanche events. At higher Rayleigh number, such as would have been characteristic of the past, the upper and lower mantle reservoirs would have been significantly more strongly disconnected chemically than they are at present. I therefore do believe that this scenario may help us immensely in reconciling the seismology with the rare gas isotope geochemistry.

S. K. Runcorn (*University of Newcastle, UK*). In a discussion of solid state convection in the Earth's mantle (Runcorn 1993) I showed, by order of magnitude calculations, that the apparently universal acceptance of the seismic tomography data as demonstrating large density anomalies over horizontal surfaces in the mantle is implausible. On this interpretation, the fractional difference in seismic wave velocity would equal the fractional difference in density multiplied by a constant about equal to 1. The volume coefficient of expansion of silicate minerals in the laboratory (about 3×10^{-5} °C^{-1}) decreases to about 10^{-5} °C^{-1} in the lower mantle, thus the roughly 1% variation of P and S wave velocities over horizontal surfaces in the mantle shown in seismic tomography maps implies variations of temperature up to 1000 °C, if thermal convection is the physical cause. Stacey (1992) also finds this implausible and appears to favour compositional differences.

The high pressure experiments on melting points that we have heard (Boehler & Jeanloz) require a temperature difference across the D'' layer of 1500 °C and Pelter's picture of plumes rising from it appears at first sight to fit the seismic tomography data as conventionally interpreted. However, the plumes would have horizontal dimensions the same as the D'' layer thickness, about 200–300 km. But

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the highest degree seismic anomalies have horizontal wavelengths of 2000–3000 km, thus the 1% density variations mapped must be the average values over these lengths. Thus if plumes are the cause, their temperatures have to be an order of magnitude greater than in Peltier's model, i.e. 10 000 °C!

Whatever the explanation of the density variations, a further problem was pointed out by Runcorn (1992): very high stress differences in the mantle result. The long wavelength variations in density of about 1% imply stress differences of up to 1000 bar in the deep mantle. At such high values solid state creep would be in the power law regime, where strain rate is proportional to the cube of stress. The effective viscosity would then be much less than the Newtonian viscosity found with stress differences of a few bars, which is what is determined from the rates of uplift found in areas where loads have been removed since the last Ice Age. This, as I pointed out (Runcorn 1993), leads to severe quantitative problems in discussion of mantle convection. Instead of density difference of 10^{-5} over horizontal surfaces, as inferred from the long wavelength parts of the geoid, this interpretation of tomography gives density differences three orders of magnitude greater. But it was shown (Runcorn 1964) that the density differences inferred from the geoid would, for viscosities of the order 10^{21} and 10^{22} poise, drive convection at the observed plate velocities (a few centimetres per year). But if the effective viscosity of power law creep is substituted and densities orders of magnitude higher are assumed, the convection velocities become absurdly high.

The discrepancies demonstrated by such order of magnitude calculation can be reduced but I do not think that they can be eliminated. I concluded (Runcorn 1993) that large scale anisotropy in the mantle, resulting from the regrowth of crystals in stress fields associated with convection, is a more likely cause of seismic velocities anomalies, for sound velocities along and across crystallographic axes are typically a few percent – of the order of those observed in the mantle. Indeed, some attempt to interpret seismic tomography in this way has been made (Montagner 1994) (though without reference to the quantitative considerations which point to this conclusion described above).

W. R. Peltier. The first point in Professor Runcorn's argument concerns the implications of seismic tomographic reconstructions of the lower mantle lateral heterogeneity insofar as temperature fluctuations are concerned. My own picture of the circulation in the lower mantle is not in fact one in which this region is seen as being inevitably dominated by isolated upwelling plumes derivative of thermal instability of the D'' boundary layer and having a horizontal scale equal to the thickness of D''. In the simulations of phase transition modulated convection (see figure 3 in this paper) the individual thermal plumes erupting from the lower boundary layer are made rather indistinct by the large enhancement of CMB heat flux that accompanies an avalanche event. During such times the thermal heterogeneity field in the lower mantle comes to be dominated by structures of extremely large horizontal scale. Furthermore, when the flow is significantly forced from within by radioactive decay processes rather than entirely from below as is the case for the example shown in figure 3, the individual upwelling plumes become an even less distinctive property of the circulation. It is not at all clear to me, therefore, that there is any problem at all with the conventional interpretation of the tomographic data as implyng temperature heterogeneity of order 10^3 °C over scales of order 2000–3000 km. The second point involves a rheological argument which also strikes me as being less than compelling. It is very well known that for fixed grain size in a polycrystalline material there exists a transition from power low creep (at low temperature) to diffusion creep (at sufficiently high temperature). Since we do not know lower mantle grain size we are in no position to argue with any assurance as to where in parameter space a given rheological mechanism may be operative. What we do know, however, is that the seismic tomographic data as conventionally interpreted do enable us to explain the large scale features of the aspherical geoid in terms of a convection model whose viscosity profile *is* compatible with that required by postglacial isostatic rebound data. It seems to me, therefore, that the picture I have developed is entirely self-consistent and not demonstrably in conflict with other lines of argument.

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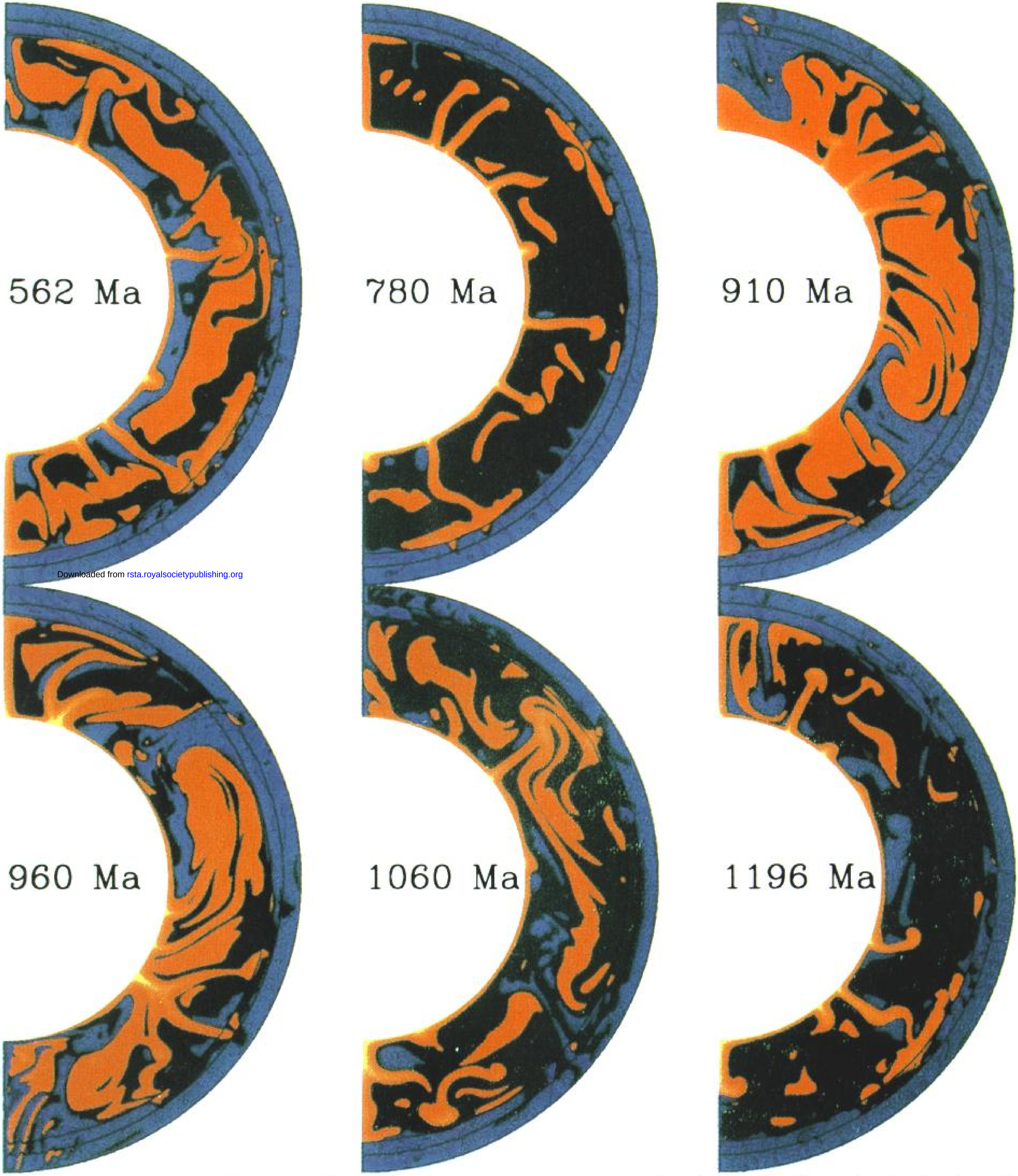

igure 3. Temperature field within the axisymmetric mantle shell at the six instants of time narked by the arrows in figure 2*c*. Note the intense cold (blue) 'avalanches' at 910 and 960 Ma hat cross the 670 (660) km discontinuity into the lower mantle. At other times the flow is very trongly layered.

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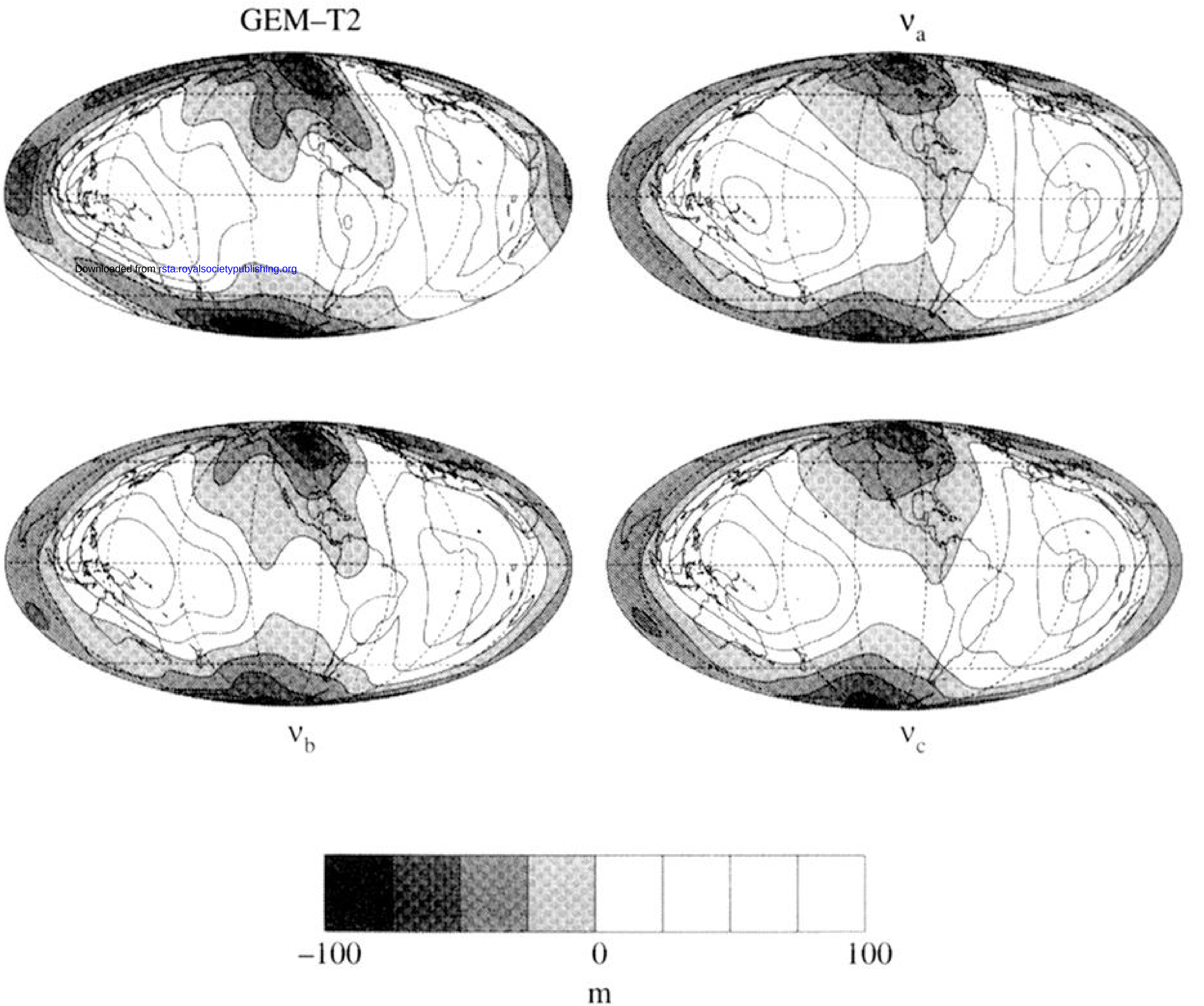

Figure 7. Observed non-hydrostatic geoid in the degree range $2 \leqslant \ell \leqslant 8$ (GEM–T2) and predicted non-hydrostatic geoids for the three viscosity models ν_a, ν_b and ν_c described in the text.

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